

The Einstein-Maxwell-aether-axion theory: Dynamo-optical anomaly in the electromagnetic response

Timur Yu. Alpin^{1,*} and Alexander B. Balakin^{1,†}

¹*Department of General Relativity and Gravitation, Institute of Physics,
Kazan Federal University, Kremlevskaya street 18, 420008, Kazan, Russia*

We consider a pp-wave symmetric model in the framework of the Einstein-Maxwell-aether-axion theory. Exact solutions to the equations of axion electrodynamics are obtained for the model, in which pseudoscalar, electric and magnetic fields were constant before the arrival of a gravitational pp-wave. We show that dynamo-optical interactions, i.e., couplings of electromagnetic field to a dynamic unit vector field, attributed to the velocity of a cosmic substratum (aether, vacuum, dark fluid...), provide the response of axionically active electrodynamic system to display anomalous behavior.

PACS numbers: 04.20.-q, 04.40.-b, 04.40.Nr, 04.50.Kd

Keywords: unit vector field, dark matter axions, dynamo-optical phenomena

I. INTRODUCTION

The term *dynamo-optical* phenomena was introduced in [1] in order to distinguish specific electromagnetic effects in media, which move non-uniformly (the macroscopic velocity field of such media is characterized by local acceleration, shear, vorticity and expansion). Irregularities of motion of electromagnetically active media can produce specific effects analogous to classical birefringence, optical activity, etc., (see, e.g., [2, 3]). A natural question arises: whether similar dynamo-optical phenomena can be displayed in *cosmic* electrodynamic systems, e.g., in pulsar magnetospheres or in super-critical black hole accretion disks? In other words, whether one has a chance to find fingerprints of cosmic dynamics in the properties of incoming light, which the astronomers are studying? The discussion of this problem is faced with a philosophic question: what is the origin of velocity field, the irregularities of which one needs to study? It might be, for instance, the velocity field, attributed to a supermassive black hole, the velocity field of a baryonic matter, of a dark matter, of a dark fluid, etc. The corresponding formalism is based on the analysis of the time-like unit eigen four-vector of the corresponding stress-energy tensor (see, e.g., [4–6]). This vector field is obtained algebraically and thus requires additional variation procedures (it was elaborated in [5, 7, 8]). An alternative approach for studying of the dynamo-optical phenomena is connected with the Einstein-Maxwell-aether theory established in [9] on the base of Einstein-aether theory [10–17]. In these theories there is a non-vanishing everywhere dynamic time-like unit vector field U^i characterizing the velocity of a cosmic substratum (the vacuum, the aether, the dark fluid and so on). The corresponding variation procedure is standard for the relativistic field theory. The Einstein-aether and the Einstein-Maxwell-aether theories realize the idea of a preferred frame of reference [18–20] associated with a world-line congruence for which the corresponding time-like velocity four-vector U^i is the tangent

vector. In this sense they are characterized by a violation of Lorentz invariance (see, e.g., [21, 22]). Based on this approach one can reformulate the idea of appearance of the dynamo-optical phenomena; now they characterize a specific type of interactions between the electromagnetic field and the unit dynamic vector field. The main detail of the mathematical description of this interaction is that the covariant derivative of the unit vector field, $\nabla_i U_k$, enters the Lagrangian linearly and in quadratic form.

In the paper [9] we mentioned three interesting applications of the established Einstein-Maxwell-aether theory; one of them is the application to the spacetimes with the so-called pp-wave symmetry. Now we consider this application in more details, using the restricted model. Generally, the full model can not satisfy the requirements of the pp-wave symmetry. Nevertheless, the Einstein-aether theory contains four Jacobson's parameters, which are not yet determined and thus can be considered as arbitrary, C_1 , C_2 , C_3 and C_4 . If we use two constraints, say, $C_2=0$ and $C_1+C_3=0$, the model, as we have shown, can accept the pp-wave symmetry. Physically, this means that the aether is considered as a substratum insensitive with respect to a shear and expansion of the velocity field, but remains sensitive to acceleration and vorticity. In this case the analysis of the model is simplified very seriously, and we used this model as an example of application. In order to explain the interest to such model, we attract the attention to the results obtained in [6], namely, that due to the dynamo-optical interactions the electric and magnetic response of the system to the action of the gravitation radiation can be anomalously strong. The model under consideration, as it will be shown below, also displays such behavior.

And the last new detail. We introduced into the Einstein-Maxwell-aether theory a pseudoscalar field [23], which could be (in principle) associated with axions and thus with the cosmic dark matter (see, e.g., [24]–[39] for details and references). Now the theory can be indicated as Einstein-Maxwell-aether-axion theory. This pseudoscalar field plays the role of mediator in the interaction between gravitational-wave, electric, magnetic and unit dynamic vector fields, providing the anomalous growth of the electromagnetic response in

*Electronic address: Timur.Alpin@kpfu.ru

†Electronic address: Alexander.Balakin@kpfu.ru

analogy with the effects described in [40]. Let us emphasize that in [40] the exact solutions to the equations of the axion electrodynamics were obtained for the case, when the background gravity field has the pp-wave symmetry, but the effects of non-uniform motion of the medium were not considered. From this point of view the presented work is the generalization of the work [40], which takes into account the dynamo-optical interactions of the electrodynamic system with the unit dynamic vector field.

The paper is organized as follows. In Section II we consider the basic elements of the Einstein-aether theory supplemented by the pseudoscalar field, and describe the background state possessing the pp-wave symmetry. In Section III we extend the model by adding the test electromagnetic field to the background field. In Section IV we obtain exact solutions for the electric, magnetic and pseudoscalar field. In Section V we discuss the anomalies in the electromagnetic response of the system on the action of the gravitational-pseudoscalar pp-waves. Conclusions are presented in Section VI.

II. BACKGROUND STATE

A. Action functional

We consider the Einstein-Maxwell-aether-axion model in the framework of a hierarchical approach. In our scheme three constituents form the background state of the global physical system: first, the gravity field; second, the unit dynamic vector field attributed to the velocity of the aether; third, the pseudoscalar (axion) field. The electromagnetic field is treated to be the test subsystem, which is influenced by the background, but does not change its state. In order to describe the background state we use the action functional

$$S_{(B)} = \int d^4x \sqrt{-g} \left\{ \frac{1}{2\kappa} [R + \lambda (g_{mn} U^m U^n - 1) + K^{abmn} \nabla_a U_m \nabla_b U_n] - \frac{1}{2} \xi \Psi_0^2 g^{mn} \nabla_m \phi \nabla_n \phi + \frac{1}{4} \Psi_0^2 \left[\frac{m_{(a)}^2}{\nu} + \nu (\phi^2 - \phi_*^2) \right]^2 \right\}, \quad (1)$$

where $g = \det(g_{ik})$ is the determinant of the metric, R is the Ricci scalar, and κ is the Einstein constant. Two last terms in this integral present the Lagrangian of a pseudoscalar (axion) field. The pseudoscalar ϕ is considered as a dimensionless quantity; the multiplier Ψ_0 brings the dimension to the pseudoscalar field. The parameter ξ takes two values. First, when $\xi = 1$, we deal with a pseudoscalar field with the fourth order potential

$$V(\phi^2) = \frac{1}{4} \Psi_0^2 \left[\frac{m_{(a)}^2}{\nu} + \nu (\phi^2 - \phi_*^2) \right]^2. \quad (2)$$

This potential possesses the following two properties:

$$V(\Phi^2) = 0, \quad \left[\frac{d}{d\phi} V(\phi^2) \right]_{|\phi=\Phi} = 0, \quad (3)$$

where the constant Φ takes one of the two values

$$\Phi = \pm \sqrt{\phi_*^2 - \frac{m_{(a)}^2}{\nu^2}}. \quad (4)$$

When $|\phi_*| > \frac{m_{(a)}}{|\nu|}$, this potential has two symmetric minima, each of them coincides with the corresponding double zero of the function $V(\phi^2)$. Second, when $\xi = -1$, the pseudoscalar field is phantom-like, or in other words, the field with negative kinetic term. Other terms in the action functional (1) involve the vector field U^i . The first term of this type, $\lambda (g_{mn} U^m U^n - 1)$, ensures that the U^i is normalized to one. The second term $K^{abmn} \nabla_a U_m \nabla_b U_n$ is quadratic in the covariant derivative $\nabla_a U_m$ of the vector field U^i , with K^{abmn} a tensor field constructed using the metric tensor g^{ij} and the velocity four-vector U^k only [10],

$$K^{abmn} =$$

$$C_1 g^{ab} g^{mn} + C_2 g^{am} g^{bn} + C_3 g^{an} g^{bm} + C_4 U^a U^b g^{mn}. \quad (5)$$

The Jacobson constants C_1 , C_2 , C_3 and C_4 are phenomenologically introduced [10–12]; there is a number of proposals to estimate these constants from observations (see, e.g., [13, 14]).

B. Master equations describing the background state

1. Equations of aether dynamics

The aether dynamic equations can be found by variation of the action functional (1) with respect to the Lagrange multiplier λ and the vector field U^i . The variation with respect to λ yields the equation

$$g_{mn} U^m U^n = 1, \quad (6)$$

which is the normalization condition of the time-like vector field U^k . The variation of the functional (1) with respect to U^i yields the dynamic equation

$$\nabla_m \mathcal{J}^{mn} - I^n = \lambda U^n. \quad (7)$$

Here we are using the standard definitions [10]

$$\mathcal{J}^{mn} = K^{lmsn} \nabla_l U_s, \quad (8)$$

$$I^n = \frac{1}{2} (\nabla_l U_s) (\nabla_m U_j) \frac{\delta K^{lsmj}}{\delta U_n} = C_4 D U_m \nabla^n U^m. \quad (9)$$

The operator D appearing in (9) is the convective derivative defined as $D \equiv U^i \nabla_i$. The Lagrange multiplier can be obtained by convolution of (7) with U_n with normalization condition (6):

$$\lambda = U_n [\nabla_m \mathcal{J}^{mn} - I^n]. \quad (10)$$

2. Equation for the pseudoscalar field

The variation of the action functional (1) with respect to pseudoscalar field ϕ gives the equation

$$\xi \nabla^k \nabla_k \phi + \left[m_{(a)}^2 + \nu^2 (\phi^2 - \phi_*^2) \right] \phi = 0. \quad (11)$$

Clearly, the constant solution $\phi = \Phi$ satisfies this equation. When $\nu = 0$ and $\xi = 1$, the equation (11) is the standard Klein-Gordon equation with the mass $m_{(a)}$.

3. Gravity field equations

The variation of the action (1) with respect to the metric g^{ik} yields the gravitational field equations

$$R_{ik} - \frac{1}{2} R g_{ik} = T_{ik}^{(U)} + \kappa T_{ik}^{(A)}. \quad (12)$$

The term $T_{ik}^{(U)}$ describes the stress-energy tensor associated with the self-gravitation of the vector field U^i ; it has the form:

$$\begin{aligned} T_{ik}^{(U)} = & C_1 (\nabla_m U_i \nabla^m U_k - \nabla_i U_m \nabla_k U^m) + \\ & + C_4 D U_i D U_k + U_i U_k U_n [\nabla_m \mathcal{J}^{mn} - I^n] + \\ & + \nabla^m [U_{(i} \mathcal{J}_{k)m} - \mathcal{J}_{m(i} U_{k)} - \mathcal{J}_{(ik)} U_m] + \\ & + \frac{1}{2} g_{ik} \mathcal{J}^{am} \nabla_a U_m, \end{aligned} \quad (13)$$

where $p_{(i} q_{k)} \equiv \frac{1}{2} (p_i q_k + p_k q_i)$ denotes symmetrization. The term

$$\begin{aligned} T_{ik}^{(A)} = & \Psi_0^2 \xi \nabla_i \phi \nabla_k \phi - \\ & - \frac{\Psi_0^2}{2} g_{ik} \left[\xi \nabla^m \phi \nabla_m \phi - \frac{1}{2} \left[\frac{m_{(a)}^2}{\nu} + \nu (\phi^2 - \phi_*^2) \right]^2 \right] \end{aligned} \quad (14)$$

describes the stress-energy tensor of the pseudoscalar (axion) field. As usual, the total stress-energy tensor satisfies the relationship

$$\nabla^k [T_{ik}^{(U)} + \kappa T_{ik}^{(A)}] = 0, \quad (15)$$

which is the identity on the solutions of (7) and (11).

C. Decomposition of the tensor $\Theta_{ik} \equiv \nabla_i U_k$

The tensor $\Theta_{ik} \equiv \nabla_i U_k$ can be decomposed, as usual, into a sum of its irreducible parts, namely, the acceleration four-vector DU^i , the symmetric trace-free shear tensor σ_{ik} , the anti-symmetric vorticity tensor ω_{ik} , and the expansion scalar $\Theta = \Theta_k^k$. The decomposition is given by

$$\Theta_{ik} = \nabla_i U_k = U_i D U_k + \sigma_{ik} + \omega_{ik} + \frac{1}{3} \Delta_{ik} \Theta, \quad (16)$$

where the basic quantities are defined as follows:

$$DU_k \equiv U^m \nabla_m U_k, \Theta \equiv \nabla_m U^m, \Delta_k^i \equiv \delta_k^i - U^i U_k,$$

$$\sigma_{ik} \equiv \frac{1}{2} \Delta_i^m \Delta_k^n (\nabla_m U_n + \nabla_n U_m) - \frac{1}{3} \Delta_{ik} \Theta,$$

$$\omega_{ik} \equiv \frac{1}{2} \Delta_i^m \Delta_k^n (\nabla_m U_n - \nabla_n U_m). \quad (17)$$

In these terms the dynamic tensor \mathcal{J}^{mn} takes the form:

$$\begin{aligned} \mathcal{J}^{mn} = & (C_1 + C_4) U^m D U^n + C_3 U^n D U^m + (C_1 - C_3) \omega^{mn} + \\ & + (C_1 + C_3) \sigma^{mn} + \frac{1}{3} (C_1 + C_3) \Theta \Delta^{mn} + C_2 \Theta g^{mn}, \end{aligned} \quad (18)$$

and the corresponding term in the action functional is

$$\begin{aligned} K^{abmn} (\nabla_a U_m) (\nabla_b U_n) = & \\ = & (C_1 + C_4) D U_k D U^k + (C_1 - C_3) \omega_{ik} \omega^{ik} + \\ & + (C_1 + C_3) \sigma_{ik} \sigma^{ik} + \frac{1}{3} (C_1 + 3C_2 + C_3) \Theta^2. \end{aligned} \quad (19)$$

D. Aether insensitive with respect to the shear and expansion of the velocity field, and solutions with pp-wave symmetry

We consider below the background model with the so-called pp-wave symmetry. This means that we assume the following:

1. The Lie derivative of the metric is equal to zero, $\mathcal{L}_{\xi_{(\alpha)}^i} g_{ik} = 0$, where the three Killing vectors $\{\xi_{(1)}^i, \xi_{(2)}^i, \xi_{(3)}^i\}$ correspond to the Abelian group of isometries G_3 , and $\xi_{(1)}^i$ is the null covariant constant Killing vector, i.e., $g_{ik} \xi_{(1)}^i \xi_{(1)}^k = 0$, and $\nabla_k \xi_{(1)}^i = 0$.

2. The vector field and pseudoscalar field inherit the pp-wave symmetry, i.e., $\mathcal{L}_{\xi_{(\alpha)}^i} U^k = 0$ and $\mathcal{L}_{\xi_{(\alpha)}^i} \phi = 0$.

Also, we consider the truncated model with $C_2 = 0$ and $C_3 = -C_1$; as it will be shown below these relationships mean that the aether does not react on the velocity perturbations, which are characterized by shear and expansion, and can answer on the acceleration and vorticity only.

Then there exist solutions to the equations for the dynamic unit vector field (7)-(10), gravity field (12)-(14) and pseudoscalar field (11), which relate to vanishing stress-energy tensors of dynamic unit vector field, $T_{ik}^{(U)} = 0$ (see (13)) and of the pseudoscalar field, $T_{ik}^{(A)} = 0$ (see (14)).

1. The proof

Space-times with the pp-wave symmetry are well-known (see, e.g. [41]) As an illustration one can use

the particular metric of this class in the so-called TT-gauge, describing, a plane gravitational wave with the first polarization [42]

$$ds^2 = 2dudv - L^2 \left(e^{2\beta} dx^2 + e^{-2\beta} dx^3 \right). \quad (20)$$

Here u and v are the retarded and advanced times, respectively, given in terms of the time t and spatial coordinate x^1 by $u = \frac{1}{\sqrt{2}}(ct - x^1)$, $v = \frac{1}{\sqrt{2}}(ct + x^1)$, and x^2, x^3 are the spatial coordinates in the plane of the front of the pp-wave. The quantities $L(u)$ and $\beta(u)$ are functions of the retarded time u only. The three Killing vectors in this case are known to be of the form

$$\xi_{(1)}^i = \delta_v^i, \quad \xi_{(2)}^i = \delta_2^i, \quad \xi_{(3)}^i = \delta_3^i, \quad (21)$$

the first of them is the null four-vector, i.e., $g_{ik}\xi_{(1)}^i\xi_{(1)}^k = 0$; also the four-vectors are orthogonal one to another.

From the requirement $\mathcal{L}_{\xi_{(\alpha)}}\phi = 0$ with the Killing vectors (21) one obtains that in the given representation the function ϕ does not depend on v, x^2, x^3 , i.e., the pseudoscalar field is the function of the retarded time u only, $\phi(u)$. The master equation for the pseudoscalar field (11) is, clearly, satisfied identically for $\phi(u) = \Phi$. The corresponding stress-energy tensor (14) vanishes, $T_{ik}^{(A)} = 0$, for arbitrary parameter ξ .

The requirement $\mathcal{L}_{\xi_{(\alpha)}}U^k = 0$ with the Killing vectors (21) means that the components U^i can be the functions of the retarded time only. We focus on the specific case of this class of solutions, namely, on the case, when

$$U^i = \frac{1}{\sqrt{2}} (\delta_u^i + \delta_v^i). \quad (22)$$

In other words, we assume that the velocity four vector has no components orthogonal to the direction of the pp-wave propagation, and the remaining components are constant in the chosen frame of reference. The covariant derivative of the velocity four-vector reduces as follows:

$$\Theta_i^k \equiv \nabla_i U^k = \frac{1}{\sqrt{2}} \left[\delta_2^i \delta_2^k \left(\frac{L'}{L} + \beta' \right) + \delta_3^i \delta_3^k \left(\frac{L'}{L} - \beta' \right) \right]. \quad (23)$$

Here and below the prime denotes the derivative with respect to the retarded time u . Clearly, the tensor Θ_{ik} based on (23) is symmetric, i.e., $\Theta_{ik} = \Theta_{ki}$; the acceleration four-vector and the vorticity tensor are equal to zero

$$DU^k = 0, \quad \omega_{pq} = 0. \quad (24)$$

The expansion scalar is proportional to the derivative of the so-called background factor L :

$$\Theta = \frac{\sqrt{2} L'(u)}{L}. \quad (25)$$

The shear tensor is also non-vanishing; it can be written as

$$\sigma_i^k = \frac{\Theta}{2} \left(\frac{1}{3} \Delta_i^k - \delta_i^1 \delta_1^k \right) + \frac{\beta'}{\sqrt{2}} (\delta_2^i \delta_2^k - \delta_3^i \delta_3^k). \quad (26)$$

Since the acceleration and vorticity is absent in the velocity field, the dynamic tensor \mathcal{J}^{mn} , obtained for the case $C_2 = 0$ and $C_3 = -C_1$

$$\mathcal{J}^{mn} = (C_1 + C_4) U^m D U^n - C_1 U^n D U^m + 2C_1 \omega^{mn} \quad (27)$$

vanishes for arbitrary constants C_1 and C_4 , i.e.,

$$\mathcal{J}^{mn}(u) = 0, \quad I^n(u) = 0. \quad (28)$$

Keeping in mind (28) and the fact that two first terms in (13) disappear, when the tensor $\Theta_{ik} = \nabla_i U_k$ is symmetric, we obtain that $T_{ik}^{(U)} = 0$.

For the case, when the unit dynamic vector field is insensitive to the shear and expansion, and the pseudoscalar field takes constant value corresponding to one of the two minima of the axion potential, the gravity field equations are reduced to the one equation

$$\frac{L''}{L} + \beta'^2 = 0, \quad (29)$$

as it should be for the model with pp-wave symmetry. We assume that the front of the gravitational pp-wave is indicated by $u = 0$, and the conditions

$$L(0) = 1, \quad L'(0) = 0, \quad \beta(0) = 0 \quad (30)$$

play the role of initial data for the metric functions.

2. Petrov's solution

For the illustration we consider the explicit Petrov solution [43]

$$L^2 = \cos ku \cdot \cosh ku, \quad 2\beta = \log \left[\frac{\cos ku}{\cosh ku} \right]. \quad (31)$$

(In fact, this solution possesses the symmetry related to the group G_5 , which includes the mentioned group G_3 as a subgroup). The corresponding interval is of the form

$$ds^2 = 2dudv - \cos^2 ku \, dx^2 - \cosh^2 ku \, dx^3, \quad (32)$$

and for this metric the Riemann tensor is covariantly constant, i.e., $\nabla_l R^i_{kmn} = 0$. Two non-vanishing components of the Riemann tensor

$$R^2_{u2u} = - \left[\frac{L''}{L} + (\beta')^2 \right] - \left[2 \frac{L'}{L} \beta' + \beta'' \right] = k^2, \quad (33)$$

$$R^3_{u3u} = - \left[\frac{L''}{L} + (\beta')^2 \right] + \left[2 \frac{L'}{L} \beta' + \beta'' \right] = -k^2, \quad (34)$$

are constants with opposite signs, providing $R_{uu} = 0$. The metric (32) is defined in the interval $0 \leq u < \frac{\pi}{2k}$, at the end of this interval, $u = u_\infty = \frac{\pi}{2k}$, one of the metric coefficients degenerates. The expansion scalar is now of the form

$$\Theta = \frac{k}{\sqrt{2}} (\tanh ku - \tan ku), \quad (35)$$

and the derivative β' (which will be necessary below) is

$$\beta' = -\frac{k}{2} (\tanh ku + \tan ku) . \quad (36)$$

The background pseudoscalar (axion) field is considered in this scheme as a constant $\phi(u) = \Phi \rightarrow \phi(0)$ and will be locally changed only in the extended model including the test electromagnetic field (see the next Section).

3. On the physical sense of the background solution

The chosen particular background model is based on a specific model of the aether, according to which the aether is insensitive to the shear and expansion of the velocity field, but can be active with respect to vorticity and acceleration. Such an aether ignores the pp-wave-type perturbations, which are characterized by the vanishing vorticity tensor and acceleration four-vector. The corresponding pseudoscalar (axion) field is constant. When we deal with test electrodynamic system and take into account the so-called dynamo-optical interactions of the electromagnetic field with the aether and the axion field, we, following the hierarchical approach, consider the space-time and the unit vector field to be non-perturbed; however, the local axion field is assumed to be changed due to the interaction with the electromagnetic field.

III. EXTENDED THEORY INCLUDING THE MAXWELL FIELD

A. Extended action functional

In [9] the Einstein-aether theory was extended by including all admissible terms with the Maxwell tensor F_{ik} . Now we consider the particular Einstein-Maxwell-aether-axion model, which is based on the action functional

$$S_{\text{(total)}} = S_{\text{(B)}} + S_{\text{(EMA)}} , \quad (37)$$

where the additional functional is of the form

$$S_{\text{(EMA)}} = \frac{1}{4} \int d^4x \sqrt{-g} \left[C_{(0)}^{ikmn} F_{ik} F_{mn} + \phi F^{ik} F_{ik}^* + X^{pqikmn} \nabla_p U_q F_{ik} F_{mn} \right] . \quad (38)$$

Here $C_{(0)}^{ikmn}$ is the linear response tensor of a test isotropic medium with dielectric and magnetic permittivities, ε and μ , respectively; this tensor can be represented in terms of the aether velocity U^i as

$$C_{(0)}^{ikmn} = \frac{1}{2\mu} [(g^{im}g^{kn} - g^{in}g^{km}) + 2(\varepsilon\mu - 1)(g^{[i}U^{n]}U^k + g^{[n}U^m]U^i)] , \quad (39)$$

where $p^{[i}q^k] \equiv \frac{1}{2}(p^iq^k - p^kq^i)$ denotes antisymmetrization. This term does not contain the

covariant derivative $\nabla_m U_n$. The tensor X^{pqikmn} describes the coupling of electromagnetic field to the non-uniformly moving aether; is was reconstructed in [9] using the metric g_{ik} , the covariant constant Kronecker tensors (δ_k^i , δ_{ab}^{ik} and higher order Kronecker tensors), the Levi-Civita tensor ϵ^{ikab} , and the unit vector field U^k itself. As in the previous case, ϕ introduces the pseudoscalar field. The asterisk denotes the dualization, $F^{*ik} \equiv \frac{1}{2}\epsilon^{ikmn}F_{mn}$ ($\epsilon^{ikmn} = \frac{1}{\sqrt{-g}}E^{ikmn}$ with $E^{0123} = 1$). As usual, we consider the Maxwell tensor expressed in terms of the electromagnetic potential four-vector A_i

$$F_{ik} = \nabla_i A_k - \nabla_k A_i , \quad (40)$$

and add the equation

$$\nabla_k F^{*ik} = 0 , \quad (41)$$

as the direct consequence of (40).

B. Electrodynamic equations

The variation of the total action functional (37) with respect to A_i gives the electrodynamic equation

$$\nabla_k H^{ik} = 0 . \quad (42)$$

Here H^{ik} is the excitation tensor linear in the Maxwell tensor

$$H^{ik} = C^{ikmn} F_{mn} , \quad (43)$$

where C^{ikmn} is a total linear response tensor

$$C^{ikmn} = C_{(0)}^{ikmn} + X^{pqikmn} \nabla_p U_q + \frac{1}{2} \phi \epsilon^{ikmn} . \quad (44)$$

This tensor includes three contributions: first, the contribution from the test medium, $C_{(0)}^{ikmn}$; second, the contribution from the dynamo-optical coupling, $X^{pqikmn} \nabla_p U_q$; third, the contribution from the axion-photon coupling, $\frac{1}{2} \phi \epsilon^{ikmn}$.

Electrodynamics of continuous media can be formulated in terms of four-vectors representing physical fields. These four-vector fields are the electric field E^i , the magnetic field \mathcal{H}^i , the electric excitation \mathcal{D}^i , and the magnetic excitation B^i [2]: They are defined in terms of F^{ik} and H^{ik} as,

$$\begin{aligned} E^i &= F^{ik} U_k , & B^i &= F^{*ik} U_k , \\ \mathcal{D}^i &= H^{ik} U_k , & \mathcal{H}^i &= H^{*ik} U_k . \end{aligned} \quad (45)$$

Completing the approach and inverting (45), we find that the tensors F^{ik} and H^{ik} can be written in terms of E^i , B^i , \mathcal{D}^i , and \mathcal{H}^i as

$$\begin{aligned} F^{ik} &= \delta_{mn}^{ik} E^m U^n - \epsilon^{ikmn} B_m U_n , \\ H^{ik} &= \delta_{mn}^{ik} \mathcal{D}^m U^n - \epsilon^{ikmn} \mathcal{H}_m U_n , \end{aligned} \quad (46)$$

where δ_{mn}^{ik} and ϵ^{ikmn} are the generalized Kronecker delta and the Levi-Civita tensor, respectively. Let us stress that now the four-vector U^i is not a observer velocity four-vector; it is the unit dynamic vector field associated with the velocity of the cosmic substratum.

C. Extended equation for the pseudoscalar field

The variation of the total action functional (37) with respect to pseudoscalar field ϕ gives now the extended equation

$$\xi \nabla_m \nabla^m \phi + \left[m_{(a)}^2 + \nu^2 (\phi^2 - \phi_*^2) \right] \phi = -\frac{1}{4\Psi_0^2} F_{ik}^* F^{ik}. \quad (47)$$

When the electromagnetic source in the right-hand side of this equation is non-vanishing, the local axion field $\phi(u)$ differs from the background constant value Φ .

D. On the dynamic equations for the aether velocity and for the gravity field

We follow the hierarchical approach, and assume that when the electrodynamic subsystem is considered to be the test one, the corresponding additional terms appeared in the equation for the aether velocity and for the gravity field are negligible in comparison with the terms, which relate to the background state. More precisely, we assume the following: first, the velocity field is non-changed and has the same form (22); second, the master equations for U^i are again satisfied identically; third, the gravity field is described again by the equation (29). In other words, we solve below the equations (41)-(44) and (47) in the given background.

E. Reconstruction of the tensor X^{pqikmn} and independent coupling constants of the dynamo-optical interactions

In order to represent the tensor X^{lsikmn} we follow definitions and the scheme used in [9]. It is well-known that the tensor of total linear response admits the decomposition

$$\begin{aligned} C^{ikmn} = & \varepsilon^{i[m} U^{n]} U^k + \varepsilon^{k[n} U^m] U^i + \\ & + \eta^{ikl} U^{[m} \nu_l^{n]} + \eta^{lmn} U^{[i} \nu_l^{k]} - \\ & - \frac{1}{2} \eta^{ikl} (\mu^{-1})_{ls} \eta^{mns}, \end{aligned} \quad (48)$$

where ε^{im} is the dielectric permittivity tensor, $(\mu^{-1})_{pq}$ is the magnetic impermeability tensor, ν_p^m is the tensor of magneto-electric coefficients:

$$\varepsilon^{im} = 2C^{ikmn} U_k U_n, \quad (49)$$

$$(\mu^{-1})_{pq} = -\frac{1}{2} \eta_{pik} C^{ikmn} \eta_{mnq}, \quad (50)$$

$$\nu_p^m = \eta_{pik} C^{ikmn} U_n = U_k C^{mkln} \eta_{lnp}. \quad (51)$$

As usual, the tensors η_{mnl} and η^{ikl} are skew-symmetric tensors orthogonal to U^i ,

$$\eta_{mnl} \equiv \epsilon_{mnls} U^s, \quad \eta^{ikl} \equiv \epsilon^{ikls} U_s, \quad (52)$$

and obey the following identities

$$-\eta^{ikp} \eta_{mnp} = \delta_{mns}^{ikl} U_l U^s = \Delta_m^i \Delta_n^k - \Delta_n^i \Delta_m^k, \quad (53)$$

$$-\frac{1}{2} \eta^{ikl} \eta_{klm} = \delta_{ms}^{il} U_l U^s = \Delta_m^i \equiv \delta_m^i - U^i U_m. \quad (54)$$

We now decompose explicitly the permittivity tensors ε^{im} , $(\mu^{-1})_{pq}$ and ν^{pm} using the non-vanishing irreducible parts of the covariant derivative of the velocity four-vector σ_{ik} and Θ . The properties

$$\varepsilon_{ik} U^k = 0, \quad (\mu^{-1})^{ik} U_k = 0,$$

$$\nu^{ik} U_k = 0 = \nu^{ik} U_i \quad (55)$$

simplify the decomposition of ε^{im} , $(\mu^{-1})_{pq}$ and ν^{pm} , providing the following results. The dielectric permittivity tensor is decomposed as

$$\varepsilon^{ik} = \Delta^{ik} (\varepsilon + \alpha_1 \Theta) + \alpha_6 \sigma^{ik}, \quad (56)$$

where $\Delta^{ik} = g^{ik} - U^i U^k$ is the projector, and α_1 and α_6 are two new independent dynamo-optical coupling constants. The magnetic impermeability tensor is decomposed as

$$(\mu^{-1})^{ik} = \Delta^{ik} \left(\frac{1}{\mu} + \gamma_1 \Theta \right) + \gamma_6 \sigma^{ik}, \quad (57)$$

where γ_1 and γ_6 are the magnetic analogs of the constants α_1 and α_6 (we use the same definitions of the coupling constants, as in [9]). The magneto-electric cross-effect pseudo-tensor is decomposed as

$$\nu^{pm} = -\phi \Delta^{pm}, \quad (58)$$

i.e., the only contribution to this tensor came from the pseudoscalar field.

Finally, to reconstruct the tensor X^{pqikmn} itself, we can put (56) and (57) into the difference $C^{ikmn} - C_{(0)}^{ikmn} - \frac{1}{2} \phi \epsilon^{ikmn}$, using (48), (39) and (58), respectively. The result is

$$\begin{aligned} X^{lsikmn} = & \frac{1}{2} \left(\alpha_1 - \frac{1}{3} \alpha_6 \right) \Delta^{ls} (g^{ikmn} - \Delta^{ikmn}) + \\ & + \frac{1}{4} \alpha_6 U_p U_q [g^{iklp} g^{mnsq} + g^{mnlp} g^{iks q}] + \\ & + \frac{1}{2} \left(\gamma_1 - \frac{1}{3} \gamma_6 \right) \Delta^{ls} \Delta^{ikmn} - \frac{1}{2} \gamma_6 \eta^{ik(l} \eta^{s)mn}, \end{aligned} \quad (59)$$

i.e., this tensor contains four new coupling constants, describing the dynamo-optical interactions. Here we used the following auxiliary tensors

$$g^{ikmn} \equiv g^{im} g^{kn} - g^{in} g^{km},$$

$$\Delta^{ikmn} \equiv \Delta^{im} \Delta^{kn} - \Delta^{in} \Delta^{km}. \quad (60)$$

IV. EXACT SOLUTIONS TO THE EQUATIONS OF EXTENDED ELECTRODYNAMICS IN THE PP-WAVE BACKGROUND

In this Section we obtain and discuss exact solutions for the electromagnetic and pseudoscalar fields, which possess the pp-wave symmetry.

A. Initial state

At $u = 0$ the metric functions satisfy the requirements (30), and the initial value for the pseudoscalar field is $\phi(0)$ (the detailed discussion concerning the Goursat problem for the pseudoscalar field can be found in [40]). The initial data for the electromagnetic field are indicated as follows: $E_v(0)$, $E_2(0)$, $E_3(0)$, $B_v(0)$, $B_2(0)$, $B_3(0)$. As for the components E_u and B_u , because of the orthogonality conditions $E_i U^i = 0$ and $B^m U_m = 0$, we obtain with $U^i = \delta_0^i$ that for arbitrary u

$$0 = E_0 = \frac{1}{\sqrt{2}}(E_u + E_v) \rightarrow E_u(u) = -E_v(u), \quad (61)$$

$$0 = B_0 = \frac{1}{\sqrt{2}}(B_u + B_v) \rightarrow B_u(u) = -B_v(u), \quad (62)$$

thus, $E_u(0) = -E_v(0)$, $B_u(0) = -B_v(0)$. Six initial parameters $E_v(0)$, $E^2(0)$, $E^3(0)$, $B_v(0)$, $B_2(0)$ and $B_3(0)$ are linked by the requirement $E_k(0)B^k(0) = 0$, which follows from the condition that the pseudo-invariant of the electromagnetic field $F_{ik}^* F^{ik}$ is equal to zero at $u = 0$, providing the compatibility of (47) at $u = 0$. In more details, this requirement reads

$$2E_v(0)B_v(0) = E^2(0)B_2(0) + E^3(0)B_3(0). \quad (63)$$

B. Exact solutions in terms of electric and magnetic fields

The first step is to solve the equations (41). Since we search for solutions inheriting the pp-wave symmetry and thus depending on the retarded time only, we can follow the simple logic way:

$$\nabla_k F^{*ik} = 0 \rightarrow$$

$$\rightarrow \frac{d}{du}(L^2 F^{*iu}) = 0 \rightarrow E^{iumn} F_{mn} = \text{const}. \quad (64)$$

Then we put $i=v$, $i=2$, $i=3$ and obtain three important formulas:

$$F_{23}(u) = F_{23}(0) \rightarrow B_v(u) = \frac{1}{L^2} B_v(0), \quad (65)$$

$$F_{2v}(u) = F_{2v}(0) \rightarrow$$

$$\rightarrow B_3(u) = e^{-2\beta} [B_3(0) + E_2(u) - E_2(0)], \quad (66)$$

$$F_{3v}(u) = F_{3v}(0) \rightarrow$$

$$\rightarrow B_2(u) = e^{2\beta} [B_2(0) - E_3(u) + E_3(0)]. \quad (67)$$

These formulas allow us to replace further the components of the magnetic field $B_v(u)$, $B_2(u)$ and $B_3(u)$ with the electric field components $E_v(u)$, $E_2(u)$ and $E_3(u)$.

Then we use the same procedure for the equation (42) and obtain

$$\nabla_k H^{ik} = 0 \rightarrow$$

$$\rightarrow \frac{d}{du}(L^2 H^{iu}) = 0 \rightarrow L^2 H^{iu}(u) = \text{const}, \quad (68)$$

or in more details

$$L^2 C^{vummn}(u) F_{mn}(u) = C^{vummn}(0) F_{mn}(0), \quad (69)$$

$$L^2 C^{2ummn}(u) F_{mn}(u) = C^{2ummn}(0) F_{mn}(0), \quad (70)$$

$$L^2 C^{3ummn}(u) F_{mn}(u) = C^{3ummn}(0) F_{mn}(0). \quad (71)$$

Using the representation of the linear response tensor C^{ikmn} (44) with $C_{(0)}^{ikmn}$ from (39) and X^{pqikmn} from (59), as well as, the formulas (65)-(67), we obtain three components of the electric field. First, we display the longitudinal (with respect to the pp-wave front) component of the electric field:

$$\Delta_{(v)}(u) \cdot E_v(u) = \varepsilon E_v(0) + B_v(0)[\phi(u) - \phi(0)], \quad (72)$$

where

$$\Delta_{(v)}(u) \equiv L^2 \left[\varepsilon + \Theta \left(\alpha_1 - \frac{1}{3} \alpha_6 \right) \right]. \quad (73)$$

Second, we display the transversal component $E^2(u)$ in the form

$$\begin{aligned} \Delta_{(2)}(u) \cdot E^2(u) &= E^2(0) \left(\varepsilon - \frac{1}{\mu} \right) + [E^2(0) + B_3(0)] \times \\ &\times \left\{ \frac{1}{\mu} (1 - e^{-2\beta}) - e^{-2\beta} \left[\Theta \left(\gamma_1 + \frac{1}{6} \gamma_6 \right) - \frac{\beta'}{\sqrt{2}} \gamma_6 \right] \right\} - \\ &- [E^3(0) - B_2(0)][\phi(u) - \phi(0)], \end{aligned} \quad (74)$$

where

$$\begin{aligned} \Delta_{(2)}(u) &\equiv L^2 \left\{ \left(\varepsilon - \frac{1}{\mu} \right) + \frac{1}{\sqrt{2}} \beta' (\alpha_6 + \gamma_6) + \right. \\ &\left. + \Theta \left[(\alpha_1 - \gamma_1) + \frac{1}{6} (\alpha_6 - \gamma_6) \right] \right\}. \end{aligned} \quad (75)$$

Finally, we obtain for the transversal component $E^3(u)$

$$\Delta_{(3)}(u) \cdot E^3(u) = E^3(0) \left(\varepsilon - \frac{1}{\mu} \right) + [E^3(0) - B_2(0)] \times$$

$$\times \left\{ \frac{1}{\mu} (1 - e^{2\beta}) - e^{2\beta} \left[\Theta \left(\gamma_1 + \frac{1}{6} \gamma_6 \right) + \frac{\beta'}{\sqrt{2}} \gamma_6 \right] \right\} +$$

$$+ [E^2(0) + B_3(0)] [\phi(u) - \phi(0)], \quad (76)$$

where the term

$$\Delta_{(3)}(u) \equiv L^2 \left\{ \left(\varepsilon - \frac{1}{\mu} \right) - \frac{1}{\sqrt{2}} \beta' (\alpha_6 + \gamma_6) + \right.$$

$$\left. + \Theta \left[(\alpha_1 - \gamma_1) + \frac{1}{6} (\alpha_6 - \gamma_6) \right] \right\} \quad (77)$$

can be obtained from $\Delta_{(2)}(u)$ by the formal replacement $\beta \rightarrow -\beta$. Thus, the electric and magnetic fields are expressed through the metric functions $L(u)$, $\beta(u)$ and their derivatives, $L'(u)$, $\beta'(u)$, and through the variation of the pseudoscalar field $[\phi(u) - \phi(0)]$.

C. Equation for the pseudoscalar (axion) field

When ϕ depends on the retarded time only, the equation for the pseudoscalar field (47) is reduced to

$$F_{ik}^* F^{ik} = -4\Psi_0^2 \nu^2 \phi [\phi^2(u) - \Phi^2], \quad (78)$$

which is, in fact, the implicit equation for the function $\phi(u)$. Using (65)-(67) this equation can be simplified as

$$\Psi_0^2 \nu^2 \phi [\phi^2(u) - \Phi^2] = -E^2(u) e^{2\beta} [B_2(0) - E^3(0)] -$$

$$-E^3(u) e^{-2\beta} [B_3(0) + E^2(0)] + \frac{2}{L^2} E_v(u) B_v(0). \quad (79)$$

Since $E^2(u)$ and $E^3(u)$ are linear with respect to $\phi(u)$, we deal with cubic equations for the axion field. We will return to this equation in the next Section.

V. ANOMALIES INDUCED BY PP-WAVES

A. Longitudinal magnetic and electric fields

In order to simplify the physical analysis of the formulas presented in general form (72)-(77), we consider separately the longitudinal and transversal configurations of the electromagnetic fields. When $E^2(0) = E^3(0) = 0$ and $B_2(0) = B_3(0) = 0$, only the longitudinal electric and magnetic fields exist in the electrodynamic system at $u > 0$. The condition (63) takes now the form $E_v(0) B_v(0) = 0$, thus, it is satisfied in two cases only.

$$1. \quad E_v(0) \neq 0, B_v(0) = 0$$

In this case we obtain from (72), (73) and (65), that

$$E_v(u) = \frac{\varepsilon E_v(0)}{L^2 [\varepsilon + \Theta (\alpha_1 - \frac{1}{3} \alpha_6)]}, \quad B_v(u) = 0. \quad (80)$$

The equation (79) yields in this case that $\phi(u) = \Phi$, i.e., the background pseudoscalar field is not disturbed.

The solution for the longitudinal electric field can grow anomalously, when the denominator in (80) tends to zero. As an illustration we consider the example of the Petrov solution and obtain

$$E_v(u) = \frac{E_v(0)}{\Delta_{||}(u) \cosh ku}, \quad (81)$$

where

$$\Delta_{||}(u) \equiv \cos ku \left[1 + \frac{k}{\sqrt{2}\varepsilon} \left(\alpha_1 - \frac{1}{3} \alpha_6 \right) \tanh ku \right] -$$

$$- \frac{k}{\sqrt{2}\varepsilon} \left(\alpha_1 - \frac{1}{3} \alpha_6 \right) \sin ku. \quad (82)$$

Clearly, $\Delta_{||}(0) = 1 > 0$ and $\Delta_{||}(\frac{\pi}{2k}) = \frac{k}{\sqrt{2}\varepsilon} (\frac{1}{3} \alpha_6 - \alpha_1)$. This means that, when $\alpha_1 > \frac{1}{3} \alpha_6$ the quantity $\Delta_{||}(\frac{\pi}{2k})$ is negative, thus, there is a moment u_{**} , for which $\Delta_{||}(u_{**}) = 0$, so that $E_v(u_{**}) = \infty$ i.e., the anomaly exists. When $\alpha_1 < \frac{1}{3} \alpha_6$ the longitudinal electric field reaches asymptotically the value

$$E_v \left(\frac{\pi}{2k} \right) = \frac{\sqrt{2}\varepsilon E_v(0)}{k (\frac{1}{3} \alpha_6 - \alpha_1) \cosh \frac{\pi}{2}}, \quad (83)$$

which is much bigger than $E_v(0)$ for small parameter $k (\frac{1}{3} \alpha_6 - \alpha_1)$. Let us stress that in this case the longitudinal electric field is finite, when the metric is degenerated.

$$2. \quad E_v(0) = 0, B_v(0) \neq 0$$

Now the longitudinal magnetic field is deformed as $B_v(u) = \frac{B_v(0)}{L^2}$, and the longitudinal electric field

$$E_v(u) = \frac{B_v(0) [\phi(u) - \phi(0)]}{L^2 [\varepsilon + \Theta (\alpha_1 - \frac{1}{3} \alpha_6)]} \quad (84)$$

is, formally speaking, generated by the axion field. The equation for the axion field (79) transforms into the following cubic equation:

$$(\phi - \Phi) \left\{ \phi^2 + \Phi \phi - \frac{2B_v^2(0)}{L^4 \Psi_0^2 \nu^2 [\varepsilon + \Theta (\alpha_1 - \frac{1}{3} \alpha_6)]} \right\} = 0. \quad (85)$$

Clearly, only one solution to this equation, $\phi(u) = \Phi$ satisfies the initial condition in general case, and we have to state that the axion field is not modified, and the longitudinal electric field can not be induced. But there is a special (critical) case, when the initial longitudinal magnetic field takes the value

$$B_v(0) = B_{\text{critical}} = \Phi \Psi_0 \nu \sqrt{\varepsilon}, \quad (86)$$

and the corresponding equation for the axion field

$$(\phi - \Phi) \left\{ \phi^2 + \Phi \phi - \frac{2\Phi^2 \varepsilon}{L^4 [\varepsilon + \Theta (\alpha_1 - \frac{1}{3} \alpha_6)]} \right\} = 0 \quad (87)$$

in addition to the constant solution $\phi(u) \equiv \Phi$ admits the solution

$$\phi(u) = \frac{1}{2}\Phi \left\{ \sqrt{1 + \frac{8\varepsilon}{L^4 [\varepsilon + \Theta(\alpha_1 - \frac{1}{3}\alpha_6)]}} - 1 \right\}, \quad (88)$$

which has the initial value $\phi(0) = \Phi$, and is of an anomalous type, when $\alpha_1 > \frac{1}{3}\alpha_6$. We deal now with a bifurcation of the solutions for the axion field.

B. The model with initially transversal pure magnetic field

In this Subsection we consider the model with initial data $B_v(0) = 0$, $E_v(0) = 0$, $E^2(0) = 0$, $E^3(0) = 0$. Clearly, in this case the longitudinal electric and magnetic field are absent for arbitrary $u > 0$.

1. Exact solutions for the transversal electric and magnetic fields

Transversal magnetic field interacting with gravitational and pseudoscalar fields generates the transversal electric field with the following components:

$$E^2(u) = \frac{B_3(0)}{\Delta_{(2)}} \left\{ \frac{1}{\mu} (1 - e^{-2\beta}) - e^{-2\beta} \left[\Theta \left(\gamma_1 + \frac{1}{6}\gamma_6 \right) - \frac{\beta'}{\sqrt{2}}\gamma_6 \right] \right\} + \frac{B_2(0)}{\Delta_{(2)}} [\phi(u) - \phi(0)], \quad (89)$$

$$E^3(u) = -\frac{B_2(0)}{\Delta_{(3)}} \left\{ \frac{1}{\mu} (1 - e^{2\beta}) - e^{2\beta} \left[\Theta \left(\gamma_1 + \frac{1}{6}\gamma_6 \right) + \frac{\beta'}{\sqrt{2}}\gamma_6 \right] \right\} + \frac{B_3(0)}{\Delta_{(3)}} [\phi(u) - \phi(0)]. \quad (90)$$

Clearly, $E^2(u \rightarrow 0) = 0$ and $E^3(u \rightarrow 0) = 0$, covering the initial data, i.e., the electric field is, indeed, generated by the gravitational wave in the axionic environment. With these formulas the magnetic field components $B_2(u)$ and $B_3(u)$ can be obtained as follows:

$$\begin{aligned} B_2(u) &= e^{2\beta} B_2(0) + L^2 E^3(u) = \\ &= \frac{L^2 B_2(0)}{\Delta_{(3)}} \left\{ \left(\varepsilon e^{2\beta} - \frac{1}{\mu} \right) + e^{2\beta} \left[\Theta \left(\alpha_1 + \frac{1}{6}\alpha_6 \right) - \frac{\beta'}{\sqrt{2}}\alpha_6 \right] \right\} - \frac{L^2 B_3(0)}{\Delta_{(3)}} [\phi(u) - \phi(0)], \end{aligned} \quad (91)$$

$$\begin{aligned} B_3(u) &= e^{-2\beta} B_3(0) - L^2 E^2(u) = \\ &= \frac{L^2 B_3(0)}{\Delta_{(2)}} \left\{ \left(\varepsilon e^{-2\beta} - \frac{1}{\mu} \right) + e^{-2\beta} \left[\Theta \left(\alpha_1 + \frac{1}{6}\alpha_6 \right) + \frac{\beta'}{\sqrt{2}}\alpha_6 \right] \right\} - \frac{L^2 B_2(0)}{\Delta_{(2)}} [\phi(u) - \phi(0)]. \end{aligned} \quad (92)$$

Formally speaking, the denominators in the formulas (89)-(92) can take zero values providing the anomalies in the responses of the electromagnetic and pseudoscalar fields. In order to clarify these possibilities in detail, we consider below an explicit example as an illustration of this anomaly.

2. Illustration of anomalous behavior on the base of the explicit Petrov's solution

We consider the formula for $E^2(u)$ only, since the results for E^3 can be obtained by the formal replacement $\beta \rightarrow -\beta$. Let us focus on the function $\Delta_{(2)}(u)$ given by (75) and appeared in the denominator of the function $E^2(u)$ (89), and let us transform it for the case, when the metric functions are given by the Petrov formulas (31):

$$\begin{aligned} \Delta_{(2)}(u) &= \cosh ku \left\{ \cos ku \left[\left(\varepsilon - \frac{1}{\mu} \right) + \right. \right. \\ &+ \frac{k}{\sqrt{2}} \tanh ku \left((\alpha_1 - \gamma_1) - \frac{1}{3}(\alpha_6 + 2\gamma_6) \right) \left. \right] - \\ &- \frac{k}{\sqrt{2}} \sin ku \left[(\alpha_1 - \gamma_1) + \frac{1}{3}(2\alpha_6 + \gamma_6) \right] \left. \right\}. \end{aligned} \quad (93)$$

At $u = 0$ and $u = \frac{\pi}{2k}$ this function takes the values, respectively,

$$\Delta_{(2)}(0) = \left(\varepsilon - \frac{1}{\mu} \right),$$

$$\Delta_{(2)}\left(\frac{\pi}{2k}\right) = -\frac{k}{\sqrt{2}} \cosh \frac{\pi}{2} \left[(\alpha_1 - \gamma_1) + \frac{1}{3}(2\alpha_6 + \gamma_6) \right]. \quad (94)$$

For the standard medium the refraction index exceeds one, i.e., $\varepsilon > \frac{1}{\mu}$, thus, $\Delta_{(2)}(0)$ is positive. When $\Delta_{(2)}\left(\frac{\pi}{2k}\right) < 0$, i.e., $\frac{1}{3}(2\alpha_6 + \gamma_6) > \gamma_1 - \alpha_1$, we deal again with the anomaly in the electric field at the moment u_{**} , in which $\Delta_{(2)}(u_{**}) = 0$. When $\frac{1}{3}(2\alpha_6 + \gamma_6) < \gamma_1 - \alpha_1$, the electric component E^2 tends asymptotically to the finite value $E^2\left(\frac{\pi}{2k}\right)$. Let us mention, that the component E^3 remains finite, when $E^2 = \infty$, and vice-versa, E^2 is finite, when $E^3 = \infty$.

The behavior of the function $\Delta_{(2)}(u)$ is more illustrative, if the unknown coupling constants are linked, e.g., by the relationship $3(\alpha_1 - \gamma_1) = \alpha_6 + 2\gamma_6$. In this case the solution u_{**} to the equation $\Delta_{(2)}(u_{**}) = 0$ can be found explicitly as

$$u_{**} = \frac{1}{k} \text{arccctg} \left[\frac{k(\alpha_6 + \gamma_6)}{\sqrt{2} \left(\varepsilon - \frac{1}{\mu} \right)} \right]. \quad (95)$$

3. Exact solutions for the pseudoscalar (axion) field

The pseudoscalar field can be found now from the equation

$$\Psi_0^2 \nu^2 \phi [\phi^2(u) - \Phi^2] = -E^2(u)e^{2\beta} B_2(0) - E^3(u)e^{-2\beta} B_3(0). \quad (96)$$

Using the definitions

$$B_2(0) = B_\perp \cos \theta, \quad B_3(0) = B_\perp \sin \theta, \quad (97)$$

we reduce this equation to the standard form of the cubic equation

$$\phi^3 + \mathcal{P}\phi + \mathcal{Q} = 0, \quad (98)$$

where the coefficients \mathcal{P} and \mathcal{Q} have the form

$$\mathcal{P} = -\Phi^2 - \frac{B_\perp^2 L^2}{\Psi_0^2 \nu^2 \Delta_2 \Delta_3} \left\{ \frac{\beta'}{\sqrt{2}} (\alpha_6 + \gamma_6) \mathcal{H}_1(u, \theta) - \mathcal{G}(u) \mathcal{H}_2(u, \theta) \right\}, \quad (99)$$

$$\begin{aligned} \mathcal{Q} = & \frac{L^2 B_\perp^2}{\mu \Psi_0^2 \nu^2 \Delta_2 \Delta_3} \left\{ \mu \Phi \frac{\beta'}{\sqrt{2}} (\alpha_6 + \gamma_6) \mathcal{H}_1(u, \theta) + \right. \\ & + \frac{\beta'}{\sqrt{2}} (\alpha_6 + \gamma_6) \left[1 - \cosh 2\beta + \mu \Theta \left(\gamma_1 + \frac{1}{6} \gamma_6 \right) \right] \sin 2\theta + \\ & \left. + \left[\left(\sinh 2\beta + \frac{1}{\sqrt{2}} \mu \gamma_6 \beta' \right) \sin 2\theta - \mu \Phi \mathcal{H}_2(u, \theta) \right] \mathcal{G}(u) \right\}. \end{aligned} \quad (100)$$

Here the auxiliary functions $\mathcal{G}(u)$, $\mathcal{H}_1(u, \theta)$ and $\mathcal{H}_2(u, \theta)$ are introduced as follows

$$\mathcal{G}(u) = \left(\varepsilon - \frac{1}{\mu} \right) + \Theta \left[(\alpha_1 - \gamma_1) + \frac{1}{6} (\alpha_6 - \gamma_6) \right],$$

$$\mathcal{H}_1(u, \theta) = \cosh 2\beta + \cos 2\theta \sinh 2\beta,$$

$$\mathcal{H}_2(u, \theta) = \sinh 2\beta + \cos 2\theta \cosh 2\beta. \quad (101)$$

At $u = 0$ the quantities \mathcal{P} and \mathcal{Q} take the values

$$\mathcal{P}(0) = -\Phi^2 + \mathcal{A}, \quad \mathcal{Q}(0) = -\mathcal{A}\Phi,$$

$$\mathcal{A} = \frac{\mu B_\perp^2 \cos 2\theta}{\Psi_0^2 \nu^2 (\varepsilon \mu - 1)}. \quad (102)$$

The corresponding cubic equation reduces at $u = 0$ to

$$[\phi(0) - \Phi][\phi^2(0) + \phi(0)\Phi + \mathcal{A}] = 0, \quad (103)$$

providing one of solutions to be $\phi(0) = \Phi$, as we need for compatibility of the model. The discriminant \mathcal{D} of the cubic equation (98), $\mathcal{D} = -(4\mathcal{P}^3 + 27\mathcal{Q}^2)$, calculated with \mathcal{P} and \mathcal{Q} , given by (99) and (100), respectively, regulates the properties of the roots of the equation

(98). It is well-known that there are three real roots, when $\mathcal{D} > 0$; two real roots from three coincide, when $\mathcal{D} = 0$; there is only one real root, when $\mathcal{D} < 0$. The corresponding Cardano formulas are well documented. Since the discriminant \mathcal{D} is a sophisticated function of retarder time and coupling parameters, we restrict ourselves by one simple case, illustrating our findings. Let the free parameters be chosen as follows:

$$\alpha_1 = \alpha_6 = \gamma_6 = 0, \quad \theta = \frac{\pi}{4}, \quad \mu = \frac{1}{\Phi}. \quad (104)$$

Then, one obtains that

$$\mathcal{Q} = 0, \quad \mathcal{P} = -\Phi^2 + \frac{B_\perp^2 \sinh 2\beta}{\Psi_0^2 \nu^2 L^2 [(\varepsilon - \Phi) - \gamma_1 \Theta]}, \quad (105)$$

and the solutions to (98) are

$$\phi_1(u) = 0,$$

$$\phi_{2,3}(u) = \pm \sqrt{\Phi^2 - \frac{B_\perp^2 \sinh 2\beta}{\Psi_0^2 \nu^2 L^2 [(\varepsilon - \Phi) - \gamma_1 \Theta]}}. \quad (106)$$

When $u = 0$ the set of these three functions covers the set of background solutions $\phi \rightarrow \{0, \pm\Phi\}$. When we deal with the Petrov metric (32), the distinguished solution with initial value $+\Phi$ behaves as follows:

$$\phi_2(u) = \sqrt{\Phi^2 + \frac{B_\perp^2 (\cosh^2 ku - \cos^2 ku)}{2\Psi_0^2 \nu^2 \mathcal{F}(u) \cos ku \cosh ku}}. \quad (107)$$

The auxiliary function $\mathcal{F}(u)$ given by

$$\mathcal{F}(u) = (\varepsilon - \Phi) \cos ku \cosh ku -$$

$$- \frac{\gamma_1 k}{\sqrt{2}} (\sinh ku \cos ku - \cosh ku \sin ku), \quad (108)$$

takes the following values at $u = 0$ and $u = \frac{\pi}{2k}$, respectively:

$$\mathcal{F}(0) = (\varepsilon - \Phi), \quad \mathcal{F}\left(\frac{\pi}{2k}\right) = \frac{\gamma_1 k}{\sqrt{2}} \cosh \frac{\pi}{2}. \quad (109)$$

This means that, when $\gamma_1 < 0$, there exists a point, $u = u_*$ in which $\mathcal{F}(u_*) = 0$, providing the value $\phi_2(u_*)$ to be infinite. When $\gamma_1 > 0$ the singularity appears at the end of admissible interval, i.e., at $u = \frac{\pi}{2k}$.

VI. DISCUSSION

The main result of the presented work is the prediction that the interaction between a unit dynamic vector field, attributed to the macroscopic velocity of a cosmic substratum (aether, vacuum, dark matter, etc.), on the one hand, and an electrodynamic system, on the other hand, can provoke an anomalous electromagnetic response on the action of a pp-wave gravitational field. In fact, we deal with the fifth model, which predicts the anomalous behavior of the electromagnetic response on the impact of the plane gravitational wave. In [44] exact

solutions were obtained, which described the anomalous response of initially static magnetic field in a simple dielectric medium at rest. In that work the amplitude of the signal-response happened to be proportional to the factor $(n^2 - 1)^{-1}$, so, the anomaly was associated with the singularity, which occurs in the vicinity of unit refraction index $n \equiv \varepsilon\mu = 1$. Such anomaly can be indicated as the static one. According to results obtained in [40], static anomaly in the vicinity of $n = 1$ exists also for the electrodynamic system surrounded by the pseudoscalar (axion) field. In [45] it was shown that in addition to the static anomaly there exists a dynamic anomaly induced by non-minimal coupling of electromagnetic and gravitational fields. This anomaly appears at some moment of the retarded time u_* , at which the denominator in the expression for the amplitude of the induced electric field takes zero value. Analogous behavior is typical for moving electrodynamically active media, when the medium motion is non-uniform (see [6]); this dynamic anomaly can be indicated as the dynamo-optical one. In this context, the results obtained in the present work generalize results, which concern both static and dynamic anomalies in the moving electrodynamic system in a pseudoscalar (axion) environment; this generalization includes couplings of the electromagnetic field to a unit dynamic vector field associated with the velocity of a distinguished cosmic substratum, e.g., aether, vacuum, etc.

In order to illustrate new results let us focus on the formula (89) supplemented by (31), (93), (107), and (108). The transversal component $E^2(u)$ of the electric field tends to infinity in two cases: first, when $\Delta_{(2)}(u) \rightarrow 0$ (see (93)); second, when $\phi_2(u) \rightarrow \infty$ (see (107), and (108)). When the coupling constants α_1 , α_6 , γ_1 and γ_6 vanish, the quantity $\Delta_{(2)}$ is proportional to $\varepsilon\mu - 1$, thus, this limit covers the results of [40, 44, 45] concerning the static anomalies in the vicinity of $n=1$.

When the mentioned coupling constants are non-vanishing, we deal with dynamic anomalies of two types. The dynamic anomaly of the first type is displayed at the moment $u = u_{**}$, for which $\Delta_{(2)}(u_{**}) = 0$ (see also the explicit formula (95) as an illustration). The amplification of the electromagnetic response is produced by the coupling of the electromagnetic field to the unit dynamic vector field, which plays the role of an energy reservoir. The dynamic anomaly of the second type appears, when $\phi(u_+) = \infty$, where u_+ is the root of the equation $\mathcal{F}(u_+) = 0$ (see (108)). Now the amplification of the electric field is mediated by the anomalous growth of the pseudoscalar (axion) field, which, in its turn, is affected by the axion-photon coupling. Again, the unit dynamic vector field plays the role of an energy reservoir for the amplification of electromagnetic and pseudoscalar (axion) fields. As in the models without pseudoscalar field mentioned above, the role of the gravitational pp-wave is provocative: it manages the process of energy redistribution between unit dynamic vector field (the reservoir) and photons (the test subsystem) in the environment of axions (the mediator).

Acknowledgments

Authors thank financial support from the Program of Competitive Growth of KFU Project No. 0615/06.15.02302.034 and from the Russian Foundation for Basic Research (Grants RFBR No. 14-02-00598 and No. 15-52-05045). ABB is grateful to Professor J.P.S. Lemos (CENTRA, IST, Lisbon) for fruitful discussions and hospitality, and acknowledges financial support provided under the European Union's FP7 ERC Starting Grant "The dynamics of black holes: testing the limits of Einstein's theory" grant agreement No. DyBHo-256667.

-
- [1] L. D. Landau, E. M. Lifshitz, and L. P. Pitaevskii, *Electrodynamics of Continuous Media*, Butterworth Heinemann, Oxford 1960, (second edition, Elsevier Butterworth Heinemann, Oxford, 1984).
 - [2] A. C. Eringen and G. A. Maugin, *Electrodynamics of Continua*, Volumes I and II, Springer-Verlag, New York, 1990.
 - [3] F. W. Hehl and Yu. N. Obukhov, *Foundations of classical electrodynamics: Charge, flux, and metric*, Birkhäuser, Boston, 2003.
 - [4] A. B. Balakin and H. Dehnen, Phys. Lett. B **681**, 113 (2009).
 - [5] A. B. Balakin and N. N. Dolbilova, Phys. Rev. D **89**, 104012 (2014).
 - [6] A. B. Balakin and T. Yu. Alpin, Gravitation and Cosmology, **20**, 152 (2014).
 - [7] A. B. Balakin, Gravitation and Cosmology, **13**, 163 (2007).
 - [8] A. B. Balakin, Class. Quantum Grav. **24**, 5221 (2007).
 - [9] A. B. Balakin and J. P. S. Lemos, Ann. Phys., **350**, 454 (2014).
 - [10] T. Jacobson and D. Mattingly, Phys. Rev. D **64** (2001) 024028.
 - [11] C. Heinicke, P. Baekler and F. W. Hehl, Phys. Rev. D **72** (2005) 025012.
 - [12] B. Z. Foster, Phys. Rev. D **73** (2006) 024005.
 - [13] C. Eling and T. Jacobson, Class. Quant. Grav. **23** (2006) 5625.
 - [14] C. Eling, T. Jacobson and M. C. Miller, Phys. Rev. D **76** (2007) 042003.
 - [15] E. Barausse, T. Jacobson and T. P. Sotiriou, Phys. Rev. D **83** (2011) 124043.
 - [16] P. Berglund, J. Bhattacharyya and David Mattingly, Phys. Rev. Lett. **110** (2013) 071301.
 - [17] T. Jacobson, arXiv:1310.5115 [gr-qc].
 - [18] C. M. Will, *Theory and experiment in gravitational physics*, Cambridge University Press, Cambridge, 1993.
 - [19] C. M. Will and K. Nordtvedt, Astrophys. J., **177** (1972) 757.
 - [20] K. Nordtvedt and C. M. Will, Astrophys. J., **177** (1972) 775.
 - [21] A. Kostelecky and M. Mewes, Phys. Rev. D **80** (2009) 015020.
 - [22] S. Nojiri and S. D. Odintsov, Phys. Rept. **505**, 59 (2011).
 - [23] W.-T. Ni, Phys. Rev. Lett. **38**, 301 (1977).
 - [24] R. D. Peccei and H. R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977).

- [25] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978).
- [26] F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).
- [27] P. Sikivie, Phys. Rev. Lett. **51**, 1415 (1983).
- [28] V. Trimble, Ann. Rev. Astron. Astrophys. **25**, 425 (1987).
- [29] M. S. Turner, Phys. Rept. **197**, 67 (1990).
- [30] G. G. Raffelt, Phys. Rept. **198**, 1 (1990).
- [31] E. P. S. Shellard and R. A. Battye, Phys. Rept. **307**, 227 (1998).
- [32] J. Silk, Lect. Notes Phys. **720**, 101 (2007).
- [33] P. Sikivie, Lect. Notes Phys. **741**, 19 (2008).
- [34] R. Battesti, B. Beltran, H. Davoudiasl, M. Kuster, P. Pugnati, R. Rabadan, A. Ringwald, N. Spooner, and K. Zioutas, Lect. Notes Phys. **741**, 199 (2008).
- [35] W.-T. Ni, Prog. Theor. Phys. Suppl. **172**, 49 (2008).
- [36] M. Khlopov, *Fundamentals of Cosmic Particle physics* (CISP-Springer, Cambridge, 2012).
- [37] A. B. Balakin, V. V. Bochkarev, and N. O. Tarasova, Eur. Phys. J. **C72**, 1895 (2012).
- [38] A. B. Balakin and L. V. Grunskaya, Rep. Math. Phys. **71**, 45 (2013).
- [39] A. B. Balakin, R. K. Muharlyamov, and A. E. Zayats, Eur. Phys. J. **C73**, 2647 (2013).
- [40] A. B. Balakin and W.-T. Ni, Class. Quantum Grav. **31**, 105002 (2014).
- [41] H. Stephani, D. Kramer, M. MacCallum, C. Hoense-laers and E. Herlt, “Exact Solutions of Einstein’s Field Equations”, University Press, Cambridge, 2003.
- [42] C. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).
- [43] A. Z. Petrov, *Einstein Spaces* (Pergamon Press, Oxford, 1969).
- [44] A. B. Balakin and D. V. Vakhruhev, Russian Physics Journal, **36**, 833 (1993).
- [45] A. B. Balakin and J. P. S. Lemos, Class. Quantum Grav. **18**, 941 (2001).